Knowability Noir: 1945-1963*

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The literature on the knowability paradox emerges in response to a modal epistemic proof first published by Frederic Fitch in his famous 1963 paper,

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"A Logical Analysis of Some Value Concepts." Theorem 5, as it was there called, threatens to collapse a number of modal and epistemic differences. Let ignorance be the failure to know some truth. Then Theorem 5 collapses a commitment to fortuitous ignorance into a commitment to necessary ignorance. For it shows that the existence of truths in fact unknown entails the existence of truths necessarily unknown. The converse of Theorem 5 is trivial (if truth entails possibility), so Fitch goes most of the way toward erasing any logical difference between the existence of fortuitous ignorance and the existence of necessary unknowability.

More exactly, it is the contrapositive of Theorem 5 that is today referred to as the *knowability paradox*. The contrapositive tells us that any truth can be known but only if every truth is in fact known. As such it collapses sophisticated anti-realism into naive idealism—a philosophical difference we may wish to preserve even if we are not sympathetic to anti-realism. Further, and with slightly strengthened resources, Fitch's proof threatens to dissolve the very distinction between what is possible and what is actual.[1]

Fitch's 1963 paper is an enigma in itself. Although much has been written about its knowability proofs, virtually nothing has been said about Fitch's understanding of their significance. That is because Fitch provides, but never comments on, the finding. Indeed, the paper appears to change subjects midway. In the first half we find some knowability proofs and gen-

[1] See Williamson (1992: 68) for the proof. What Williamson shows, precisely, is this: $\Box(p \leftrightarrow \Diamond Kp) \vdash_{T+K} \Diamond p \leftrightarrow p$, which says that if, necessarily, a proposition is true just in case it is knowable, then it follows in modal system T (augmented with minimal epistemic resources) that a proposition is possible just in case it is true.

eral lessons about concepts that share certain logical properties with the concept of knowledge. In the second half we find a logical analysis of a particular concept of value, which happens not to share the relevant logical properties with the concept of knowledge. Why does Fitch develop and include the knowability results in a paper whose primary goal is to articulate a logical analysis of value? It initially appears that the knowability considerations have nothing to do with Fitch's final analysis.

The thesis of the present paper is that Fitch's intent was to pinpoint a disruptive set of logical properties that lend themselves to the trivialization of conditional analyses. Or, at the very least, Fitch included the central theorems to demonstrate a sort of conditional fallacy that threatens, although not irredeemably, against his own analysis of value. If this is right, then Fitch does not take the knowability proofs to be paradoxical, but instead takes them to be a lesson about how intensional operators interact, surprisingly, to thwart the efforts of conditional analyses. Fitch's demonstration of the knowability proofs may be understood as a logical lesson in how to avoid the so called "conditional fallacy" in philosophical analysis.

My reading of Fitch is based on unpublished papers archived at Yale, Columbia and Princeton. The important documents include a pair of reports from 1945 (Edition, *This Volume*), in which an anonymous referee conveyed to Fitch the knowability proof. The handwriting of the draft to the editor gives away its author, which is unmistakably Alonzo Church. The subsequent debate between Fitch and Church paints a clearer picture of what Fitch, by 1963, perceived to be the philosophical significance of the so-called paradox of knowability. The archival documentation puts us in a position, for the first time, to articulate and evaluate a lost chapter in the history and philosophy of logic—the early history of the knowability paradox.

The 1963 Paper: What's He Building in There?

The published literature begins with Fitch's 1963 paper. Here Fitch investigates intensional operators that are factive and closed under conjunction-elimination. An operator O is factive just when its application implies truth:

(Factivity) $\Box(Op \rightarrow p)$

The formula says, necessarily, if Op then p. Factive operators include 'it is true that,' 'it is known that,' 'it is perceived that,' and 'it is necessary that.' By contrast, 'it is believed that' is not factive, since believing p does not require the truth of p.

An operator is closed under conjunction-elimination (or is conjunction-distributive) just when it applies to a conjunction only if it applies to the corresponding conjuncts:

(&-E Closure) $\Box(O(p \,\&\, q) \rightarrow (Op \,\&\, Oq))$

Both knowledge and belief are conjunction-distributive in this sense, since knowing/believing a conjunction requires knowing/believing each of the conjuncts. Fitch's concern primarily is with 'knows,' which is both factive and conjunction-distributive.[2]

[2] There are few exceptions to the received view that 'knows' is both factive and conjunction-distributive. Robert Nozick (1981), for instance, articulates a concept of knowledge that is not conjunction-distributive.

Fitch's concern in the first half of the paper is only with operators that satisfy these two principles and the theorems in which they figure. He proves six theorems. Their content is discussed below. The philosophical significance of each theorem, if any, I for now leave open, since Fitch did not comment on their significance.

The first two theorems are perfectly general. I paraphrase the first:

> Theorem 1: for any factive propositional operator O that is closed with respect to &-E,
> $\neg\Diamond O(p \,\&\, \neg Op)$.

Fitch proves here that there is always an *un-O-able* proposition, when O has the aforementioned logical properties. The proof is well rehearsed in the literature for the case of knowledge. Substituting the knowledge operator K for O gives us a theorem about the unknowability of any Fitch-conjunction, $p \,\&\, \neg Kp$. The unknowability may be stated this way: $\neg\Diamond K(p \,\&\, \neg Kp)$.

The demonstration follows:[3]

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{K(p \;\&\; \neg Kp)}\,(1)}{Kp \;\&\; K\neg Kp}\,(\text{\&-E Closure})}{Kp \;\&\; \neg Kp}\,(\text{Factivity \& trivial logic})}{\neg K(p \;\&\; \neg Kp)}\,(1)}{\neg\Diamond K(p \;\&\; \neg Kp)}\,(\text{Normal Modal Logic})
$$

At the top of the tree we suppose for reductio that the Fitch-conjunction, $p \,\&\, \neg Kp$, is known. By the closure of knowledge under &-E, it follows

[3]The Genzen-Prawitz notation is preferred throughout for the perspicuity of logical dependencies.

that each conjunct is known. The third line demonstrates an application of factivity to the right conjunct of the second line. In the face of the ensuing contradiction, we discharge and deny our only assumption. By necessitation and the duality of the modal operators, we conclude with the impossibility of that assumption. Fitch-conjunctions are unknowable! And more generally, conjunctions of the form $p \;\&\; \neg Op$ are un-O-able, when O is factive and conjunction-distributive.

Fitch's second perfectly general theorem says this: for the aforementioned operators, O, if p is a true proposition that is not O-ed, then $p \;\&\; \neg Op$ is a true proposition that is un-O-able.

> Theorem 2: for any factive operator O that is closed under &-E, if p is true but un-O-ed, then that it is an un-O-ed truth is itself un-O-able.
>
> $(p \;\&\; \neg Op) \rightarrow \neg\Diamond O(p \;\&\; \neg Op)$.

The result follows trivially from Theorem 1. The remainder of the theorems, Theorems 3 through 6, are special cases or consequences of the above perfectly general results.

> Theorem 3: If an agent a is all-powerful in the sense that anything that is true could have been brought about by a, then everything that is true was brought about by a:
>
> $\forall p(p \rightarrow \Diamond aBp) \rightarrow \forall p(p \rightarrow aBp)$.

B is the factive, conjunction-distributive operator 'brought it about that.' Theorem 3 follows from Theorem 1, substituting B for the operator variable, O.

The next two theorems are the knowability proofs. Theorem 4 is credited by Fitch to an anonymous referee.

> Theorem 4: for each agent that is not omniscient, there is a true proposition that that agent cannot know:
> $\exists p(p \,\&\, \neg aKp) \rightarrow \exists p(p \,\&\, \neg\Diamond aKp)$.

Theorem 4 is the contrapositive of Theorem 3, replacing the knowledge operator, K, for B.

The next theorem is Theorem 5. It or its contrapositive is most often equated with the knowability paradox. It is a modification of Theorem 4.

> Theorem 5: If there is a true proposition which nobody knows (or has known or will know) to be true, then there is a true proposition which nobody can know to be true:
> $\exists p(p \,\&\, \forall a \neg aKp) \rightarrow \exists p(p \,\&\, \forall a \neg\Diamond aKp)$.

Theorem 5 strengthens both the antecedent and the consequent of Theorem 4. It does this by generalizing over subjects in both places. Theorem 5 is then slightly more interesting when we detach the consequent, since it commits us to the existence of a truth that cannot be known *by anyone*. When we suppress the quantifiers ranging over subjects, as is standardly done for ease of exposition, Theorems 4 and 5 say the same thing—viz., if there is an unknown truth then there is an unknowable truth. Although the referee conveyed to Fitch Theorem 4 and thereby informed this entire section of Fitch's paper, the slightly more interesting Theorem 5 (i.e., the so-called knowability paradox) and the perfectly general theorems are owed, at least in part, to Fitch.

Fitch's final result, Theorem 6, just is Theorem 5, replacing 'knows that' with 'proves that' and stipulating that our propositions p are themselves about proving.

> Theorem 6: If there is some true proposition about proving that nobody has proved or ever will prove, then there is some true proposition about proving that nobody can prove:
> $\exists p(p \ \& \ \forall a \neg aPp) \rightarrow \exists p(p \ \& \ \forall a \neg \Diamond aPp)$, where our propositional variables range over propositions about proving.

The set of six theorems in their own right constitute an interesting development in the logic of intensional operators and action, and they play a role in current developments of modal epistemic logic.[4] Fitch, as we mentioned, does not comment on their significance. If the first half of Fitch's 1963 paper is about the logic of un-O-ability, then what is its connection to the apparently unrelated subject that occupies Fitch in the second half of the paper?

The second half of the paper is concerned to articulate a logical analysis of an informed-desire theory of value. *Informed-desire* says, roughly, that something is valuable to a subject just when she would desire it if she had all the relevant information.[5] Fitch's final analysis in 1963 roughly is this: s *values* p just when there is a truth q, such that, necessarily, if s knows that q then s strives for p. The logical analysis appears as *Definition 6*:

[4]See for instance Rescher (2005) and van Benthem (*This volume*).

[5]The counterfactual gloss appears in an earlier draft of the paper (1961) and is meant to capture a causal reading. Fitch borrows from the logical analysis of causation found in William Burks (1951). In so doing, Fitch (1961: 6a) explains that the relevant sense of 'A causes B' is strict implication in a modal system such as S2 or M.

(D6) Vp iff $\exists q(q \,\&\, \Box(Kq \rightarrow Sp))$.[6]

D6 is the centerpiece of the 1963 paper, but Fitch develops analyses of other propositional operators as well. He offers, for instance, definitions of 'knows,' 'does,' 'can do,' and 'desires.' In each of these cases he employs the strict (causal) conditional in the analysis and ends with considerations about whether or not the main operator (or definiendum) is factive and conjunction-distributive. It is only in the case of his causal definition of knowledge, D2, that we find an operator that is *both* factive and conjunction-distributive. The main attraction, i.e., the analysis of value, however, is conjunction-distributive but not factive.

Importantly, 'Knows', which, again, is the only factive, conjunction-distributive operator defined in the second half of the paper, figures in Fitch's definition of value. So, a desideratum for understanding Fitch is this: *the significance of the knowability theorems must carry a lesson about the role played by 'knows' in Fitch's analysis of value.* But which lesson? Why in a paper about how to articulate an informed-desire theory is Fitch concerned to prove the knowability results? The question can be answered more carefully once we have uncovered the lost history of the proofs. So we leave this section with the central question, to which we will return. What's he building in there?

[6]Some liberty is taken here with the formalism. Fitch uses 'C' for '(partially) causes' rather than the necessary conditional, although it is clear from the text and from the 1961 address that a strict conditional reading is adopted. (See previous note.) Moreover, there are epicycles in Fitch's analysis that involve other propositional variables. Not being relevant to the present discussion, they are suppressed.

Who Discovered Fitch's Paradox?

Another curiousity of Fitch's 1963 paper is the identity of the famous anonymous referee, to whom Fitch credits the first of the two knowability results. Following Theorem 4, Fitch tells us,

> . . . This theorem is essentially due to an anonymous referee of an earlier paper, in 1945, that I did not publish. This earlier paper contained some of the ideas of the present paper. (1963: 138, n.5)

That is all that Fitch says on the matter. The present section reveals more.

We find that Fitch's 1945 paper was titled "A Definition of Value" and submitted to the *Journal of Symbolic Logic* in January or February of 1945.[7] And although so many recent papers mention the anonymous referee (under that description), few have published speculation about his identity. According to Richard Routley (Sylvan),

> [Robert] Meyer conjectures, what seems to me unlikely, that Anon[ymous] = Gödel. (1981: 110, n. 12)

Routley's skepticism is not explained. Why think that Gödel is an unlikely suspect for authorship of the result? Gödel was not officially an editorial consultant for *JSL* in 1945. More interestingly, as John Burgess noted to me, it was unlikely that the editors would have asked Gödel to referee a paper, since his perfectionism would have prevented him from returning a report in a timely manner.

[7] The title is mentioned in Nagel (1945b).

As for Robert Meyer, he recently admitted that he does not recall having ever discussed Fitch's paradox with Routley, but notes that he "would have been struck by the strong whiff of the Gödel formula,"[8] which says of itself that it is true but unprovable. Meyer is recalling the first incompleteness result, which demonstrates that, for any consistent, sufficiently strong theory T in the language of arithmetic, there are truths unprovable in T. For any such theory T, we find that there is a sentence p such that p is true but unprovable in T:

$$p \ \& \ \neg P_T p.$$

The resemblance to the anomalous Fitch-conjunction, $p \ \& \ \neg Kp$, catches one's attention here. The Gödel-conjunction and the Fitch-conjunction are analogous epistemic claims. Both advocate that the truth of some proposition p cannot be established by certain means. An important difference of course lies, first, in the self-reference that is indicative of the Gödel sentence and, second, in the epistemic terminology. For Gödel the terminology is "unprovable in T." For Fitch the notion is, less formally but more generally, "unknowable." Gödel promises a truth that could never be proven in T. Fitch promises a truth that could never be known by any means.

Wolfgang Künne (2003: 425, n. 159) briefly considers the hypothesis that Gödel was the originator of the knowability result but notes that Fitch's result is "in one respect more ambitious" than Gödel's theorem. Künne's suggestion, I believe, is a claim about the relative logical strength of the respective claims to unknowability. Gödel shows us that there is a truth

[8] *Personal Correspondence.*

that cannot be proven in T, but of course this does not entail that the truth could not be proven by some other means. Whereas, via Fitch we may conclude that there is a truth, viz., the Fitch-conjunction, $p \,\&\, \neg Kp$, that is unknowable, full stop. On the contingent assumption that $p \,\&\, \neg Kp$ is true, it does follow by Fitch's result that $p \,\&\, \neg Kp$ is an unknowable truth. And so, it follows that it could not be proven in *any* consistent theory strong enough for arithmetic.

The problem with taking the Fitch conclusion to be logically stronger is that the existence of unknowable truths depends on the existence of some ignorance, which arguably is a contingent matter. However, some have contended that the existence of some ignorance is logically necessary.[9] If it is necessary, then the conclusion of the knowability result is in fact stronger than Gödel's first incompleteness theorem.

The Gödel-hypothesis is the only candidate in the literature. In the Summer of 2005, however, an altogether different hypothesis emerges. The hypothesis was prompted by found correspondence between the 1945 coeditors of *JSL*. In a letter dated March 6, 1945 Ernest Nagel updates Alonzo Church:[10]

> I made a copy of your report on Fitch's ms. (on the assumption that his receiving your handwritten version would destroy your anonymity) and sent it to him with the statement that his ms. in its present form was not acceptable for publication by the JSL.

[9] See, for instance, Routley (1981) and Rescher (2005: Appendix 2).

[10] Thanks to John Burgess for his assistance in searching the Alonzo Church Papers and for identifying this letter. Thanks to Herbert Enderton for bringing the Church Papers to my attention.

> He replied two days later – I enclose his letter; and yesterday he returned the ms. with another letter appended. I do not think he has met either of your two fundamental objections – indeed, his reply to the second difficulty seems to me to evade the issue rather completely. I am sending you the material for any further comments you may wish to make. (Nagel 1945a)

The letter indicates a number of things. In 1945 Church refereed a paper written by Fitch; the author of the report was anonymous to Fitch; and Fitch's paper was (at least, at this stage) not being accepted for publication. The evidence is circumstantial, but if this was the paper in question and there were no other referees on the job, then it would seem that Church was the anonymous referee who conveyed the knowability proof to Fitch in 1945.

The Nagel letter led me to the Ernest Nagel Papers (Columbia University), where my research assistant, Julien Murzi, very quickly identified the referee report in October 2005. The document had not previously been identified. It was composed in Church's trademark vertical handwriting, and thereby confirmed that Church indeed was the referee. In the excerpt below we find the earliest known formulation of the knowability proof. Church writes,

> ...it may plausibly be maintained that if a is not omniscient there is always a true proposition which it is empirically impossible for a to know at time t. For let k be a true proposition which is unknown to a at time t, and let k' be the proposition that k is true but unknown to a at time t. Then k' is true. But

> it would seem that if a knows k' at time t, then a must know k at time t, and must also know that he does not know k at time t. By Def.2, this is a contradiction.[11] (1945a; Report 1, p.2)

In sum, if a person a is not omniscient (that is, if there is a truth unknown to a), then there is a truth unknowable to a. It is evident that this result becomes the first knowability result, Theorem 4—the very result that Fitch credits to the anonymous referee.

It is not surprising that Church was the author of the report and its main proof, which is often taken to be about the logical limits of knowledge. It would be understated to say that Church thought deeply about such matters. He formalized the concept of effective calculability (1936, AJM) and proved the undecidability of first-order logic (1936, JSL).

Possible influences on Church's thought in 1945 include Gödel's work from the prior decade and the interactions the two philosophers had in Princeton in the years leading up to 1945. In *JSL* William Parry (1939: 140) had proved in Theorem 22.8: $\neg\Diamond\neg\Diamond(p \rightarrow \neg\Diamond\neg p)$, which is equivalent to $\neg\Diamond\Box(p \,\&\, \neg\Box p)$— the core of Theorem 4, replacing all occurrences of $\Box$ with K. There was also Moore's Paradox (1942: 543), which reveals the peculiarities of propositions of the form, 'p but I don't believe p.'[12]

The critical documents found in the Nagel Papers actually include two referee reports, which we will label chronologically, Reports 1 and 2 (or R1 and R2). They are included in their entirety as the first essay of this volume.

[11] Church refers here to Def. 2, which appears to be Fitch's definition of knowledge. As can be seen from the context, Church employs it to exploit the factivity of knowledge.

[12] Thanks to Roy Sorensen for info about this related problem and its earliest source.

Report 2 was written by the same hand as Report 1. The second, but not the first, report was signed by the author. The originals were apparently seen only by Nagel in his capacity as editor, who typeset them to preserve Church's anonymity. In large part they consist of a series of trivialization arguments against Fitch's analysis of value. Some of these arguments utilize the knowability result quoted above. The next section evaluates the ideas central to Report 1.

The First Referee Report

A Trivialization of Fitch's Analysis

The knowability result was developed by Church to trivialize Fitch's analysis of 'a values p at time t,' which is referred to in Report 1 as '$Def.3$.' No statement of $Def.3$ appears in the report, but the context allows us to reconstruct the definition as follows:

(Def. 3) $Vp \;\; iff \;\; \exists q(q \,\&\, \Box(Kq \rightarrow Dp))$

The formula tells us that it is valued (or is valuable to a subject) that p just when there is a truth q, such that knowing q necessarily implies desiring p.[13]

[13]From Church's report we learn that Fitch employs a notion of 'empirical necessitation' rather than 'strict implication' in the right-hand side of the definition and distinguishes between the two notions throughout his paper. Fitch's strict implication is Lewis and Langford's. The modality is governed by S2. Fitch's empirical necessitation, by contrast, appears to be a weaker notion. At the very least, Fitch's Th. 1 appears to be a principle stating that strict implication entails empirical necessitation. See for instance the application of Th. 1 in Report 2, page 1. I suppress Fitch's distinction between strict implication and empirical necessitation throughout. The issues here do not hang on the decision.

The analysis tells us, for instance, that it is valuable to me that I take my migraine medication if it is true that the medication will stop the pain and knowing that it stops the pain leads me to desire that I take the medication.

Church's criticism of the analysis begins with the acknowledgment that we are non-omniscient—that there are some truths p that an agent a does not know (at time t). Formally, for some p, it is true that

(1) $p \mathbin{\&} \neg Kp$.

By the familiar result it is impossible for a to know both that p is true and that p is not known by a (at t).

(2) $\neg\Diamond K(p \mathbin{\&} \neg Kp)$.

Conditionals with impossible antecedents are necessarily true. So, from (2), it follows that

(3) $\Box(K(p \mathbin{\&} \neg Kp) \rightarrow r)$, where r is any proposition you like.

Let r be 'It is desired by a that s' or just 'Ds'. Then

(4) $\Box(K(p \mathbin{\&} \neg Kp) \rightarrow Ds)$.

Hence, from (1) and (4) it follows that there is a truth q such that knowing q strictly implies desiring s:

(5) $\exists q(q \mathbin{\&} \Box(Kq \rightarrow Ds))$.

Therefore, by $Def.3$, s is valued:

(6) Vs.

And since s was arbitrarily chosen, it therefore follows that everything is valued. In sum, if there is a truth unknown to a then a values everything.

At a glance the result is this, where q is the true conjunction $p \,\&\, \neg Kp$.

$$\dfrac{\dfrac{q \qquad \dfrac{\overline{\neg\Diamond Kq}}{\Box(Kq \to Ds)}}{\exists q(q \,\&\, \Box(Kq \to Ds))}}{Vs}\ \text{(Def.3)}$$

Church's argument illustrates the mistake in Fitch's analysis. The mistake tends to occur when we define concepts in conditional terms. This, the so called "conditional fallacy," is not unrelated to the paradoxes of implication. Classical conditionals behave strangely when their antecedents are false or impossible. More specifically, but without attempting to characterize all and only cases of the fallacy, the conditional fallacy is a mistake that occurs just when the antecedent of the conditional definiens is not always logically independent of the definiendum. That is, instances of the analysis include cases where the definiendum contradicts, entails or is entailed by the antecedent of the conditional definiens. Such conditions will sometimes effect a surprising disparity in truth value between the definiens and the definiendum.[14]

This is what gets Fitch's analysis into trouble. The conditional embedded in his definition, $Vp \;\; iff \;\; \exists q(q \,\&\, \Box(Kq \to Dp))$, has instances where the antecedent, Kq, is not logically independent of the definiendum, Vp. And that is because there are instances of the antecedent that are logically impossible and so entail any proposition whatsoever. *A fortiori*, such instances necessarily imply the definiendum.

[14]This understanding of the fallacy is informed by Shope (1978) and Wright (2000).

The mistake in Fitch's analysis results from his failure to detect the logical anomaly of unknowable truth. For the existence of unknowable truth is the logical phenomenon responsible for the surprising trivialization of Fitch's analysis. Fitch later takes to heart this lesson of philosophical analysis. The lesson will play a critical role in Fitch's 1963 paper.

In the second referee report Church considers blocking the above trivialization by appealing to Russell's theory of types. In so doing Church foreshadows Linsky (*this volume*) and Hart (*this volume*). However, Church dismisses the option as contrary to Fitch's purposes, since an employment of the theory of types would invalidate closure principles central to Fitch's paper. As we will see in the next section, Church has independent reason for rejecting these closure principles.

Closure Principles for Knowledge and Belief

In the first report Church foreshadows what he takes to be Fitch's only good defense against the trivialization argument, and that is to question the validity of closure principles for propositional attitude operators. Specifically, he denies that there is a "law according to which one who believes a proposition must believe all its logical consequences" (Report 1: 2). Church questions here the validity of the principle that belief is closed under logical consequence. His intention, though, is to question the justification for the principle that belief is closed under conjunction-elimination. Church writes,

> To be sure, one who believes a proposition without believing its more obvious logical consequences is a fool; but it is an empirical fact that there are fools. It is even possible that there might be

> so great a fool as to believe the conjunction of two propositions without believing either of the two propositions; at least an empirical law to the contrary would seem to be open to doubt. On this ground it is empirically possible that a might believe k' at time t without believing k at time t (although k' is a conjunction one of whose terms is k).[15]

Church denies that belief is necessarily closed under conjunction elimination. It is unclear, however, how this is supposed to help Fitch. The trivialization argument never utilizes a closure principle for belief. It utilizes, instead, a closure principle for knowledge. And, of course, it would be a fallacy of division to suppose that the concept of belief has a certain logical property P (e.g., closure under logical consequence) just because (1) belief is a component of knowledge and (2) knowledge has P.[16]

An alternative, non-fallacious reading of Church's passage is that he simply means "knowledge" when he speaks of belief. In that case Church simply questions whether knowledge is closed under conjunction-elimination. However, this limited closure principle is harder to reject than the more general principle that knowledge is closed under logical consequence. That is because "knowing p" and "knowing q" are implicit in "knowing p & q." So it is doubtful that Church has offered Fitch a "good defense" of the trivialization argument. But let us suppose that he has and consider one further point about the denial of closure in this context.

[15] R1: 2-3.

[16] More recent instances of this very closure-fallacy in epistemology are detected by Ted Warfield (2004).

Directly after his articulation of the problems with taking belief to be closed under conjunction-elimination and offering the defense on Fitch's behalf, Church makes the following claim:

> Unfortunately this defense compels Fitch to abandon his Ax. 1. And, what is more serious, it lights the way to a second and opposite objection to Def. 3.
>
> If there is no empirical law according to which one who believes a proposition must believe its logical consequences, it would seem that by the same token there is no empirical law according to which a person's desires must be in reasonable accord with that person's beliefs. (R1: p. 3)

The consequence of rejecting closure principles for belief, according to Church, is that it invites a skepticism about other principles that express necessary connections between propositional attitudes, in particular between knowledge and desire. And without such necessary connections, the right-hand side of Fitch's analysis is never satisfied, and so, the theory is trivialized in the opposite direction. Nothing is of value to anyone!

Church's point is overstated. Surely there may be laws about our propositional attitudes, even if belief/knowledge is not closed under logical consequence more generally. That is, for all we know, some principles other than the unrestricted closure principles justify necessary connections between our propositional attitudes. Fitch, in fact, gives the following example in reply to Church: necessarily, if it is known that I desire that p, then I desire that p. So sometimes knowledge does necessitate desire. The example is an instance of the principle that knowledge necessarily implies truth. With it Fitch proves trivially that there are some necessary connections between knowledge and desire. Fitch's example is cited in Church's second referee report. (R2: 4)

We learn from Nagel's letter of March 6 that Fitch replied to the first referee report with two letters and a revised manuscript. These documents, like Fitch's initial submission, are yet to be found. In any case Nagel was unimpressed by them. Recall Nagel's remark to Church: "I do not think [Fitch] has met either of your two fundamental objections—indeed, his reply to the second difficulty seems to me to evade the issue rather completely. I am sending you the material for any further comments you may wish to make." The second difficulty, recall, was Church's animadversions to closure principles and other "laws" relating propositional attitudes. I do not see that Fitch's point—about factively knowing that one desires something—evades Church's difficulty "completely," but I will not pursue the issue further. Fitch's very revealing reply to the other difficulty, i.e., the trivialization argument from unknowable truth, is summarized in the second referee report. We turn to that document next.

The Second Report

Fitch's Cartesian Restriction Strategy

In reply to Nagel's March 6 letter, Church issued a second referee report. From it we learn of Fitch's reactions to the trivialization argument in Report 1. Fitch's analysis said that it is valuable that p just in case there are truths that would, if known, necessitate the desire that p. Church showed us that, vacuously, there will be such truths, since there are truths that it is impossible to know. The natural reply is to restrict the class of truths to those that it is possible to know. Presumably it is only the knowable truths that should figure in causal relations between knowledge and desire. In reply to the first report, Fitch endorses this insight by offering an alternative restricted theory of value, Def. 3R:

(Def. 3R) Vp iff $\exists q(q \mathbin{\&} \Diamond Kq \mathbin{\&} \Box(Kq \rightarrow Sp))$[17]

The restricted analysis says that something p is of value to a subject a just when there is some *knowable* truth q that would, if known, necessitate a's desiring that p. Let us call this 'Fitch's *Cartesian* restriction strategy,' because it foreshadows Neil Tennant (1997), where the restriction is proposed under that name to block the knowability paradox. Tennant defines a Cartesian proposition p as one for which Kp is not provably inconsistent. Tennant (2001) considers versions of the restriction in terms of what it is *metaphysically* possible to know. The Cartesian restriction on the relevant class of truths blocks the problematic unknowable truth, '$p \mathbin{\&} \neg Kp$,' from consideration. The fact that knowledge of it vacuously implies an arbitrary proposition becomes inconsequential.

[17] The ammended analysis appears in Report 2: 2. The above formulation of Fitch's Def. 3R differs from Church's in that I substitute $\Diamond$ for 'EP', which reads, 'it is empirically possible that.' Also, I continue to replace 'EN' or 'empirically necessitates' with the necessary material conditional and drop the variables ranging over subjects.

Church's Objection to the Cartesian Restriction Strategy

In the second report Church announces that "a reductio ad absurdum of Def 3R is possible along the same lines as that I have given for Def 3." Church claims there is a Cartesian truth that trivializes Fitch's restricted analysis. He begins by noting that, for some p, p is an unknown truth. So

(i) $Dp' \vee (p \;\&\; \neg Kp)$

is true, for an arbitrary proposition p'. That is, (i) follows from our non-omnscience. After all, if $p \;\&\; \neg Kp$ is true, then so is the weaker claim, $Dp' \vee (p \;\&\; \neg Kp)$. Church goes on to argue that

(ii) Proposition (i) is Cartesian.

We shall evaluate this and the next premise in a moment. The final premise is that knowing proposition (i) strictly implies the desire that p':

(iii) $\Box(\; K(Dp' \vee (p \;\&\; \neg Kp)) \rightarrow Dp' \;)$

If premises (i), (ii) and (iii) are all correct, then it follows that there is a Cartesian truth q such that, necessarily, if q is known then p' is desired. By Fitch's Cartesian restricted theory of value, Def. 3R, it would follow that p' is valuable, for arbitrary p'. We may generalize. If an agent is non-omniscient, then everything whatsoever is valuable to her!

Here we consider the premises of Church's argument. Premise (i) is trivial. If $p \;\&\; \neg Kp$ is true for some p, then so is $Dp' \vee (p \;\&\; \neg Kp)$, by disjunction-introduction. What about premise (ii)? It says that the awkward disjunction, given by (i), is Cartesian, i.e., can be known. Church begins his defense of this premise with the reasoning that any desire is possible (Report 2: pp. 2-3):

(a) for any p', $\Diamond Dp'$.

But then knowledge of that desire is possible:

(b) for any p', $\Diamond K(Dp')$.

And so, by the closure of knowledge under disjunction-introduction,

(c) for any p', $\Diamond K(Dp' \vee (p \,\&\, \neg Kp))$.

Church defends premise (a) by noting that anything, even something crazy like one's instant death, can be desired, since it is possible to be insane or in a position less fortunate than one brought about by instant death. By similar reasoning, we might argue further that even contradictions can be desired. So an arbitrary proposition can be desired. But how does Church get from premise (a) to premise (b)? He seems to be assuming that, necessarily, any possible desire is a knowable desire. That is, if it is possible for one to desire that p' then it is possible for one to know that one desires that p'. Note that there are some implicit principles being invoked. We uncover them by asking what it takes to justify the principle that any possible desire is a knowable desire? Perhaps Church believes that, necessarily, any desire can be known. So, necessarily, if p' is desired, then it is possible to know that p' is desired:

$$\Box(Dp' \rightarrow \Diamond K(Dp'))$$

On this reading, Church assumes an unrestricted knowability principle about desire. Notice that this is not sufficient to license the move from line (a) to line (b). For the assumption that it is possible to desire p', together with the above principle, by minimal normal modal reasoning, entails only that it is possible that it is possible that Dp' is known:

$$\Diamond\Diamond K(Dp').$$

The S4 axiom is then needed to reduce this to $\Diamond K(Dp')$. So if the operant notion of possibility satisfies the S4 axiom and in fact, necessarily, any desire is knowable, then it follows that, necessarily, any possible desire is a knowable desire. With this latter principle in hand, premise (b) does in fact follow from premise (a).

Premise (c) then follows from premise (b) on the assumption that knowledge is closed under disjunction-introduction.

$$Kp \vdash K(p \vee q)$$

The principle, Church tells us, "seems to be entirely in the spirit of [Fitch's] Th. 3." (Report 2: 3) Th. 3 we may hypothesize to be the principle stating that knowledge is closed under conjunction-elimination. Church puts these principles on a logical par. Presumably he is onto the fact that both are instances of the principle that knowledge is closed under *obvious* logical consequence.

There is, however, the objection that these two closure principles are not on a par. 'a knows both that p and q' and 'a knows p and a knows q' are implicit in one another. Arguably, they say the same thing. Such gives us reason to think that knowledge is closed under conjunction-elimination, despite the problems with thinking that knowledge is closed more generally under logical consequence. By contrast, 'a knows p' and 'a knows $p \vee q$' are not each implicit in one another. The latter is not implicit in the former, since q may embed concepts that are not grasped by one who understands 'a knows p'. So it is not decisive that knowledge should be closed under disjunction-introduction, even if it is closed under conjunction-elimination.

Now Fitch never commits himself to the S4 axiom, and need not be committed to the closure of knowledge under disjunction-introduction, even if he does accept its closure under conjunction-elimination. So Church's argument for premise (ii) is not decisive. His logical assumptions are not trivial.[18]

We turn to Church's justification for premise (iii). How does Church prove that $\Box(\ K(Dp' \vee (p \mathbin{\&} \neg Kp)) \rightarrow Dp')$? The reasoning here is troubling. It can be found on page 3 of Report 2. Church notes that, by the factivity of knowledge, knowing $Dp' \vee (p \mathbin{\&} \neg Kp)$ entails $Dp' \vee (p \mathbin{\&} \neg Kp)$. And further that each of these disjuncts implies Dp'. So, by proof-by-cases, Dp'. Therefore, if $Dp' \vee (p \mathbin{\&} \neg Kp)$ is known, then Dp'. Resting on no contingent assumptions, $Dp' \vee (p \mathbin{\&} \neg Kp)$ necessarily implies Dp'. Here is the reasoning at a glance:

[18]Incidently, there is a more modest defense of premise (ii). That is, it is metaphyiscally possible to know premise (i) for the following reason. It is possible to know Dp' , recognize that Dp' entails $Dp' \vee (p \mathbin{\&} \neg Kp)$, and thereby come to know $Dp' \vee (p \mathbin{\&} \neg Kp)$. If this is right, then it is after all possible to know $Dp' \vee (p \mathbin{\&} \neg Kp)$. That is, premise (i) is Cartesian.

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{K(Dp' \lor (p \mathbin{\&} \neg Kp))}\,(2)}{Dp' \lor (p \mathbin{\&} \neg Kp)} \qquad \dfrac{}{Dp'}\,(1) \qquad \dfrac{\dfrac{}{p \mathbin{\&} \neg Kp}\,(1)}{Dp'}}{Dp'}\,(1)}{K(Dp' \lor (p \mathbin{\&} \neg Kp)) \to Dp'}\,(2)}{\Box(\ K(Dp' \lor (p \mathbin{\&} \neg Kp)) \to Dp')}
$$

If this is correct then there is a Cartesian truth, such that knowing it necessitates desiring p', for any proposition p'. By Fitch's Cartesian restricted theory of value, therefore, p' is valued. The theory trivializes.

If this is Church's argument, then we have to reject it. Something went wrong in the proof-by-cases. The right disjunct $p \mathbin{\&} \neg Kp$ does not imply Dp'. Church may be confusing the proposition $p \mathbin{\&} \neg Kp$ with $K(p \mathbin{\&} \neg Kp)$. The latter strictly implies everything, since it is impossible. So obviously $K(Dp' \lor (p \mathbin{\&} \neg Kp))$ strictly implies Dp'. Perhaps that is what Church intended, and the mistake can be chalked up to a misprint.

However, with the supposition of a misprint in the formulation of Church's proof-by-cases one must make corresponding adjustments to the first part of Church's argument. The truth that must be shown to be Cartesian is now $Dp' \lor K(p \mathbin{\&} \neg Kp)$. It is Cartesian. It is knowable because its left disjunct is. But it is not true. Or at least it is not true for an arbitrary desire, Dp'. Therefore, the trivialization argument comes apart; it fails against Fitch's Cartesian restricted theory of value.[19]

[19] Jim Stone (in *personal correspondence*) constructs an argument for premise (iii) in Church's spirit. It presupposes the transparency of desire (i.e., that all desires are known, $Dp \to K(Dp)$, and all failures to desire are known, $\neg Dp \to K(\neg Dp)$. It also presupposes that knowledge is closed under disjunctive syllogism. It goes like this. Suppose $K(Dp' \lor (p \mathbin{\&} \neg Kp))$, and suppose for reductio that $\neg Dp'$. By the transparency of desire, $K(\neg Dp')$.

Other items that appear in the second referee report include (1) a more formal (Lewis and Langford style) proof of the central knowability result that appeared in the first report; (2) some mention of the similarity of the trivialization arguments to the liar and set-theoretic paradoxes and the standard devices for resolving them; (3) a brief mention of a problem of accepting the factivity of knowledge while embracing a theory of types; (4) further discussion of Fitch's concept of empirical necessity; and (5) some counterexamples to Fitch's theory of value that do not hinge on the knowability theorems. I will not comment on these items.

Fitch does not seem to have directly addressed Church's final trivialization argument against the Cartesian restricted theory. In Nagel's last letter to Church on the matter (April 13, 1945), we learn that Fitch has withdrawn his paper owing to "a defect in my definition of value" and because "the paper should be rewritten anyhow."

Fitch in the 60s

The 1945 Church-Fitch debate helps to explain some things about the 1963 paper. One question is about the intended significance of the knowability results. Why does Fitch include them? Consider again the knowability the-

Since a disjunction and the negation of the left disjunct are both known, it follows, by the closure of knowlege under disjunctive syllogism, that the right disjunct is known—giving $K(p \,\&\, \neg Kp)$. But that is impossible. So by classical reductio, Dp'. Hence, by conditional proof, $K(Dp' \vee (p \,\&\, \neg Kp)) \rightarrow Dp'$. So a defense of premises (iii) may be given and Church's master argument may be rehabilitated, although the text does not warrant crediting this argument to Church. An even more modest master argument against Fitch's restricted analysis is formulated in the final section of this paper.

orems, which say, roughly, that there is an unknowable truth if there is an unknown truth. Fitch presents them in passing but does not comment on their significance. Of course, that he demonstates the proofs without comment indicates that he takes them to be valid and not paradoxical. Moreover, it is obvious that there are unknown truths, and so, by the relevant theorems, it would seem that we are meant to recognize the existence of unknowable truths. The insight is intrinsically interesting, but the question regards its role in the paper.

One interpretation is that Fitch is offering a refutation of verificationism, the thesis that all meaningful statements (and so, all truths) are verifiable. Indeed, this is how the early literature interprets Fitch.[20] To be consistent, this reading requires us to argue, analogously, that there are implicit conclusions that Fitch wishes us to draw from the other theorems. Recall that Theorem 3 shows that if there is an omnipotent being, then he has in fact done everything. Presumably, Fitch would expect us to conclude from this that there is no omnipotent being (or that he is not supremely good, or that there is no free will, or something of the sort). However this is an unlikely reading of Fitch's intent, as it marks the theorems, including the knowability proofs, as a curious tangent from the paper's primary goal. The 1963 paper is not a defense of any metaphysical position, not even a defense of the informed-desire theory of value. Rather, it aims to articulate the logical content of that theory. It would be odd, in a paper with that purpose, for Fitch to prove the absurdity of verificationism or disprove the existence of

[20]See, for instance, Hart and McGinn (1976); Hart (1979); Mackie (1980); and Routley (1981).

God. More to the point, this interpretation of the theorems tells us nothing about the factive, conjunction-distributive role played by 'knows' in Fitch's analysis of 'value.' So it would seem that the key to understanding Fitch lies elsewhere.

'Knows', unlike the other concepts that Fitch defines in the second half of the 1963 paper (including the concept of value), is both factive and conjunction-distributive. For this reason it gives rise to the existence of unknowable truth. Fitch wishes us to recognize the existence of unknowable truths for logical, not metaphysical reasons. Unknowable truth is the hallmark of the kind of trivialization that Fitch wishes to avoid in 1963—the very kind of trivialization that Church wielded against him in 1945. Through the 1945 exchange Fitch recognizes that conditional analyses harbor grave pitfalls. When the dominant propositional operator of the antecedent of a conditional definition is factive and conjunction-distributive, then there will be an instance of the conditional analysis whose antecedent is impossible. But then the antecedent will not be logically independent of the definiendum (whatever it is), and consequently, trivialization threatens. For the special case, the moral of the knowability theorems, is then to beware of this fallacy in the conditional understanding of the informed-desire theory.

My explanation of why Fitch included the knowability theorems in the paper is supported by the fact that Fitch does in fact heed the warning by protecting against the fallacy. Directly following the formal articulation of his 1963 analysis of value, $Vp \quad \text{iff} \quad \exists q(q \;\&\; \Box(Kq \rightarrow Sp))$, Fitch explains that to avoid absurdity,

> q may be regarded as containing all the *knowable* relevant infor-

mation.(1963: 142, *my emphasis*)[21]

We see here the very Cartesian restriction that Fitch attempted in 1945, although Fitch includes it here without much remark. It appears then that the reason that the knowability theorems are included in the first half of the paper is to explain the need for the Cartesian restriction that emends the final analysis in the second half.

Recall that in 1945 Fitch attempted in this way to restrict his analysis in reply to Church's first referee report. But the attempt was met with an overwhelmingly negative second report. At the time Fitch decided to withdraw his paper, even though the second report, as we have seen, was critically flawed. However, Fitch must have recognized the errors of Church's second report. For by 1963 he was perfectly happy with a Cartesian-restriction in his final analysis of value.

Why did Fitch wait so long to publish the analysis? I believe that skepticism about non-trivial necessary connections between knowledge and desire kept Fitch sufficiently worried, and that it was not until Burks (1951) offered a logical analysis of causal conditionals that Fitch believed himself to have the logical resources to explain the relevant modal relation. This is supported by the fact that Fitch makes extensive use of Burk's analysis in both his presidential address to the Association for Symbolic Logic (1961) and his 1963 publication.

We have seen that the role of the knowability theorems in Fitch's paper do carry lessons about the role played by 'knows' in Fitch's final analysis of value. These are the aforementioned lessons about whether and how to pro-

[21]See other mentions of the restriction on page 141.

tect against the conditional fallacy. *In 1963 what Fitch is building in there is an analysis of value that is sheltered from this fallacy.* With this interpretation of the knowability proofs, we find an account of the early history and initial, perceived significance of the so-called knowability paradox.

Against Fitch's Cartesian Restriction

Church's result shows us that there are unknowable truths, and that these truths serve to trivialize Fitch's 1945 theory of value. Fitch responds by restricting his theory to truths that it is possible to know. The problem with the restriction, as Church attempted to show in his second report, is that there are *knowable* truths that trivialize the restricted theory. The trick is to come up with a knowable truth q, such that q is weaker than the unknowable truth, $p \mathbin{\&} \neg Kp$, and such that q trivializes the theory. Church's choice of such a proposition was $Dp' \vee (p \mathbin{\&} \neg Kp)$. It is knowable, but as I argued it fails to do the job that Church set for it. And that is because knowing that proposition does not necessitate an arbitrary desire. Church fails to trivialize the restricted theory, but he was right in thinking it can be done. There are in fact knowable truths weaker than $p \mathbin{\&} \neg Kp$ that serve to trivialize Fitch's restricted theory of value. They are truths of the following form

(1) $p \mathbin{\&} (Kp \rightarrow q)$

which says both that p and that knowing p implies q. That will be true whenever there is an unknown truth—i.e., whenever

(2) $p \mathbin{\&} \neg Kp$

is true. So whenever $p \,\&\, \neg Kp$ is true for some sentence p, $p \,\&\, (Kp \rightarrow q)$ will be true for an arbitrary sentence q. And that is because a false proposition (in this case Kp) materially implies any proposition.[22] Therefore, for some proposition p, the following is true:

(3) $p \,\&\, (Kp \rightarrow Sq)$.

Moreover, knowledge of its truth necessarily implies that q is strived for:

(4) $\Box(K(p \,\&\, (Kp \rightarrow Sq))) \rightarrow Sq$.[23]

And finally, $p \,\&\, (Kp \rightarrow Sq)$ is Cartesian. That is, it is possible to know $p \,\&\, (Kp \rightarrow Sq)$, even though it is not logically possible to know the logically stronger proposition, $p \,\&\, \neg Kp$.

In sum, if there is an unknown truth p, then $p \,\&\, (Kp \rightarrow Sq)$ is a knowable truth, and, necessarily, knowing it necessitates striving for q. But then there is a knowable truth, p, that satisfies $\Box(Kp \rightarrow Sq)$. Consequently, the right-hand side of Fitch's theory of value is vacuously true. It follows by the restricted theory of value that q is valued, for arbitrary q. The theory collapses!

The formalism below more perspicuously demonstrates the collapse of Fitch's 1963 theory of value. After all, $p \,\&\, (Kp \rightarrow Sq)$ is a knowable

[22]Such consequences of the Fitch-conjunction are used by Williamson (2000: 110-112) and Brogaard and Salerno (2006: 266-267) against Tennant's (1997) Cartesian restriction strategy. Interesting discussion also appears in Rosenkranz (2004), although his prescription is for the Cartesian restriction strategist to reject normal modal logic.

[23]The proof of (4) is straightforward. It requires that K be factive and closed under conjunction-elimination.

proposition; knowing it necessarily implies Sq; and it is true if $p \,\&\, \neg Kp$ is true, for some p.

$$\cfrac{\cfrac{\cfrac{\cfrac{p \,\&\, \neg Kp}{p \,\&\, (Kp \to Sq)} \qquad \cfrac{}{\Box(K(p \,\&\, (Kp \to Sq)) \to Sq)}}{(p \,\&\, (Kp \to Sq) \,\&\, \Box(K(p \,\&\, (Kp \to Sq)) \to Sq)}}{\exists q(q \,\&\, \Box(Kq \to Sq))}\ (\mathrm{D6})}{\cfrac{Vq}{\forall q Vq}}$$

Although the above argument trivializes Fitch's theory of value, it does not uncover a conditional fallacy. The conditional's antecedent $K(p \,\&\, (Kp \to Sq)$ is logically independent of the definiendum Vq. There is, however, a conditional fallacy that the 1963 analysis perpetrates. This is demonstrated by a different version of the above argument. Just replace all occurrences of the formula $p \,\&\, (Kp \to Sq)$ in the above proof with

(*) $\quad p \,\&\, (Kp \to (Vq \,\&\, Sq))$.

(*) appears to be Cartesian; knowing it is not logically independent of the definiendum, Vq; knowing (*) strictly implies Sq; and it succeeds in trivializing the theory, since Sq and its embedded proposition, q, were chosen arbitrarily.

There are instances of Fitch's definition of value where the antecedent of the relevant conditional is not logically independent of the definiendum. If a lesson of Church's 1945 result is not to commit the conditional fallacy in philosophical analysis, then by 1963 Fitch had appreciated the danger but his analysis had not satisfactorily protected against it.

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